TRINITY COLLEGE

Year 11 Mathematics Specialist Units 1&2
Test 6   2022

Calculator Free
Proof & Complex Numbers

STUDENT'S NAME _____

DATE: Friday 14th October   **TIME:** 45 minutes   **MARKS**: 48

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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Questions begin on the next page.

1.   (7 marks)

Given $w = 3 - 5i$ and $z = i - 2$, evaluate the following:

(a) $2w + z$ [1]

(b) wz [2]

(c) $\overline{wz}$ [1]

(d) $\dfrac{w}{z}$ [3]

2.   (3 marks)

Determine the complex number w if $w + iw = 1 + 7i$.

3.   (3 marks)

Express $0.03\overline{4}$ as a rational number.

4.   (6 marks)

Prove, by contradiction, $\sqrt{7}$ is irrational.

5.   (4 marks)

Determine all exact solutions (real and complex) for the equation $x^3 - 4x^2 + 13x = 0$

6.   (6 marks)

Prove by mathematical induction, $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1-r^n)}{1-r}$, $r \neq 1$.

7. (6 marks)

Consider the expression $m^2 + 7$

(a) Evaluate the expression $m^2 + 7$ for $m = 1, 3, 5, 7$ and 9 [1]

(b) Use your values from (a) to state the largest integer, p, that $m^2 + 7$ is always divisible by, when m is a positive odd integer. [1]

(c) Prove that $m^2 + 7$ is always divisible by p when m is a positive odd integer. [4]

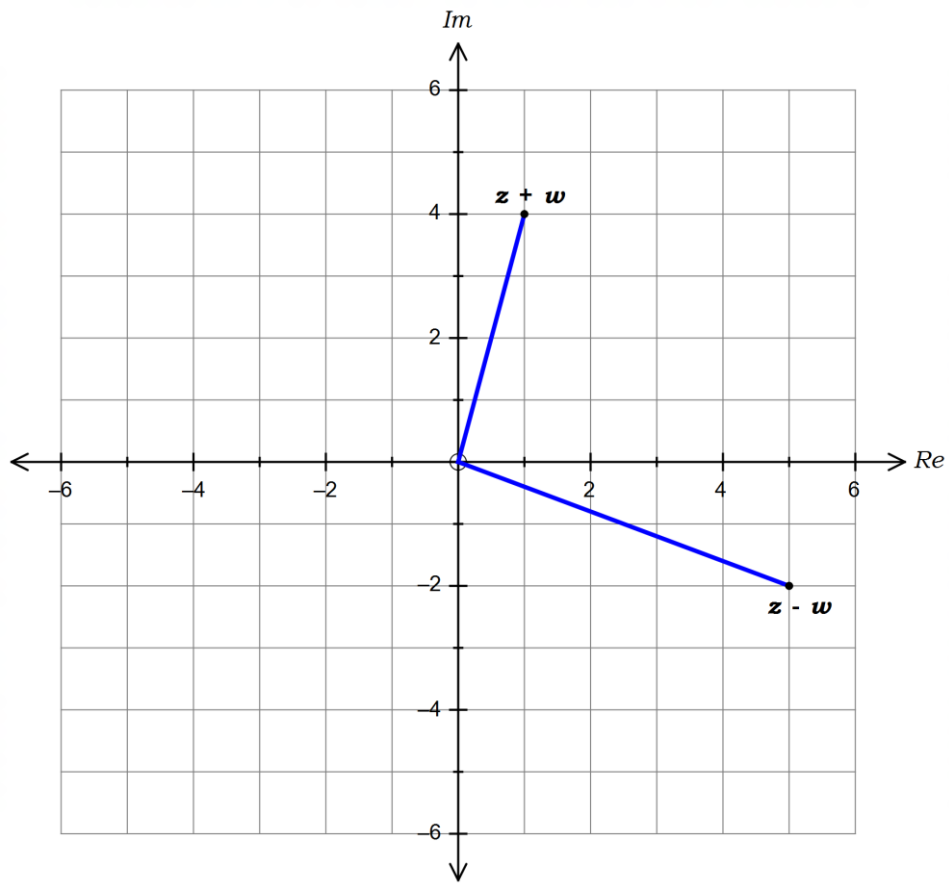8.   (13 marks)

(a)   Given that $p^n = -i$, where $n \in \mathbb{Z}$, determine each of the following:

(i) p^{n+1} [2]

(ii) $(p^n - p^{-n})^2$ [3]

(b) Two complex numbers w and z are such that their addition and subtraction are shown on the diagram below.

Add and label each of the following to the grid above.

(i) $zi + wi$ [2]

(ii) $\dfrac{z - w}{i}$ [2]

(iii) z [2]

(iv) w [2]

Additional Working Space:

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